HOW SHARP IS THE SHARPE-RATIO? - RISK-ADJUSTED PERFORMANCE MEASURES

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How sharp is the Sharpe-ratio? - Risk-adjusted Performance Measures

"Alpha to omega, downside to drawdown, appraisal to pain"

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Any discussion on risk-adjusted performance measures must start with the grandfather of all risk measures the Sharpe Ratio[1] or *Reward to Variability* which divides the excess return of a portfolio above the risk free rate by its standard deviation or variability:

Sharpe Ratio $SR = \dfrac{r_P - r_F}{\sigma_P}$

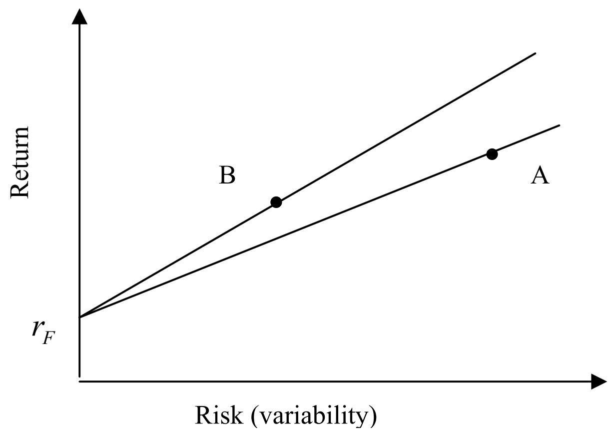
Where:

r_P = portfolio return normally annualised

r_F = risk free rate (annualised if portfolio return is annualised)

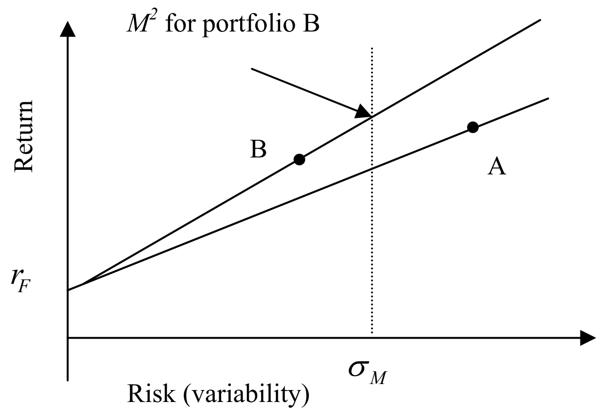
σ_P = portfolio risk (variability, standard deviation of return) again annualised if portfolio return is annualised

Most risk measures are best described graphically, a measure of return in the vertical axis and a measure of risk in the horizontal axis as shown below:



Ideally if investors are risk averse they should be looking for high return and low variability of return, in other words in the top left-hand quadrant of the graph. The Sharpe ratio simply measures the gradient of the line from the risk free rate (the natural starting point for any investor) to the combined return and risk of each portfolio, the steeper the gradient, the higher the Sharpe ratio the better the combined performance of risk and return.

The Sharpe ratio is sometimes erroneously described as a risk-adjusted return; actually it's a ratio. We can rank portfolios in order of preference with the Sharpe ratio but it is difficult to judge the size of relative performance. We need a risk adjusted return measure to gain a better feel of risk-adjusted outperformance such as M^2 shown below.



A straight line is drawn vertically through the risk of the benchmark σ_M. The intercept with the Sharpe ratio line of portfolio B would give the return of the portfolio with the same Sharpe ratio of portfolio B but at the risk of the benchmark. This return is called M^2, a genuinely risk adjusted return, extremely useful for comparing portfolios with different levels of risk. It is relatively straight forward to calculate:

$$M^2 = r_P + SR \times (\sigma_M - \sigma_P)$$

Where:

$\sigma_M =$ market risk (variability, standard deviation of benchmark return)

The statistic is called M^2 not because any element of the calculation is squared but because it was first proposed by the partnership of Leah Modigliani and her grandfather Professor Franco Modigliani[2]. Variability can be replaced by any measure of risk and M^2 calculated for different types of risk measures. I prefer this presentation of the statistic; it clearly demonstrates there is a return penalty for portfolio risk greater than benchmark risk and a reward for portfolio risk lower than the benchmark risk. Those more familiar with Modigliani's work would recognise the following formula, although the answer is still the same:

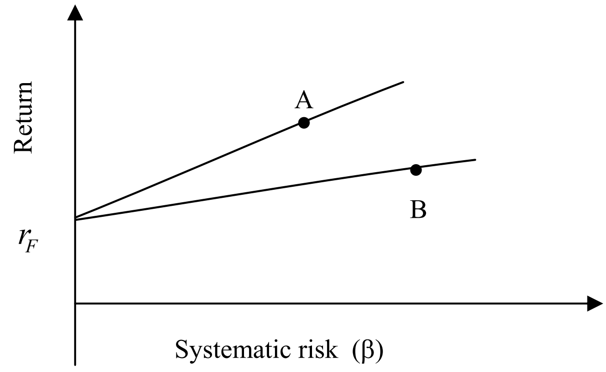
$$M^2 = (r_P - r_F) \times \frac{\sigma_M}{\sigma_P} + r_F$$

Investment statistics can either be grouped as Sharpe type combining risk and return in a ratio, risk adjusted returns such as M^2 or descriptive statistics which are neither good nor bad but provide information about the pattern of returns.

The regression statistics β (or systematic risk), ρ (correlation), covariance and R^2 (or correlation squared) are descriptive statistics. Jensen's alpha is often misquoted as the portfolio manager's excess return above the benchmark, more accurately it is the excess return adjusted for systematic risk.

Treynor ratio or *reward to volatility* is similar to Sharpe ratio, the numerator (or vertical axis graphically speaking) is identical but in the denominator (horizontal axis) instead of total risk we use systematic risk as calculated by *beta.*

Treynor Ratio $= \dfrac{r_P - r_F}{\beta_P}$



Treynor ratio is extremely well know but perhaps less frequently used because it ignores specific risk. If a portfolio is fully diversified with no specific risk the Treynor and Sharpe ratios will give the same ranking. Some academics favour the Treynor ratio because they believe any value gained from being not fully diversified is transitory. Unfortunately the performance analyst does not have the luxury of ignoring specific risk when assessing historic return.

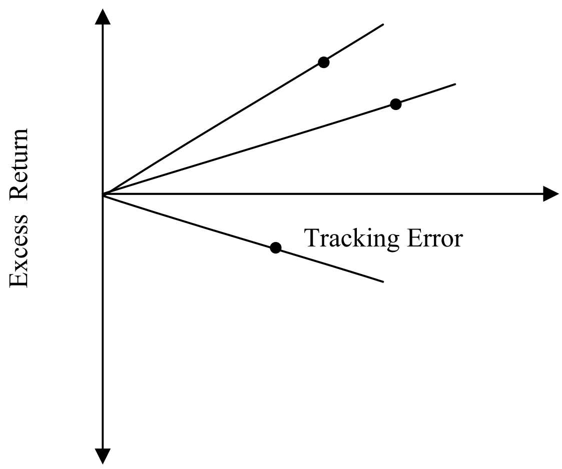
The appraisal ratio first suggested by Treynor & Black[3] (1973) is similar in concept to the Sharpe ratio but using Jensen's *alpha,* excess return adjusted for systematic risk in the numerator (vertical axis), divided by specific risk not total risk in the denominator (horizontal axis).

$$\textbf{Appraisal Ratio} = \frac{\alpha}{\sigma_{\varepsilon}}$$

This measures the systematic risk adjusted reward for each unit of specific risk taken. Although seldom used I must say this statistic appeals to me and perhaps should be given more consideration by investors

In exactly the same way we compared absolute return and absolute risk in the Sharpe ratio you can compare excess return and tracking error (the standard deviation of excess return) in the information ratio

The information ratio is similar to the Sharpe ratio except that instead of absolute return on the vertical axis we have excess return, and instead of absolute risk on the horizontal axis we have tracking error or relative risk, the standard deviation of excess return.



We have no need for a risk free rate since we are dealing with excess returns; the information ratio lines always radiate from the origin. The gradient of the line is simply the ratio of excess return and tracking error as follows:

$$\textbf{Information Ratio}\ \ \mathrm{IR} = \frac{\text{Annualised Excess Return}}{\text{Annualised Tracking Error}}$$

A negative information ratio is an indication of underperformance, smaller magnitude negative information ratios indicating a better combined performance than larger magnitude negative information ratios. Some commentators, Israelsen[4] (2005), consider this anomalous, because higher tracking errors generate better results and suggest modifying the information ratio to ensure high tracking errors are always penalised. This is of course a nonsense, it self-evident to me at least, that if you are going to underperformance it is far better to inconsistently underperform (high tracking error) than consistently underperform (low tracking error). The information ratio requires no modification

The Sharpe, appraisal, Treynor and information ratios are familiar measures used by the industry for decades; they take the familiar form of reward divided by risk. More recently hedge funds have encouraged the use of additional risk measures designed to accommodate the risk concerns of different types of investors. These measures can be categorised as based on normal measures of risk, regression, higher or lower partial moments, drawdown or value at risk (Va*R*) as follows:

Type	**Combined Return and Risk Ratio**
Normal	Sharpe & Information, Modified Information
Regression	Appraisal & Treynor
Higher or lower partial moments	Sortino, Omega, Upside Potential, Omega-Sharpe & Prospect
Drawdown	Calmar, Sterling, Burke, Sterling-Calmar, Pain & Martin
Value at Risk	Reward to VaR, Conditional Sharpe, Modified Sharpe

Not all distributions are normal distributed, if there are more extreme returns extending to the right tail of a distribution it is said to be positively skewed and if they are more returns extending to the left it is said to be negatively skewed

We can measure the degree of skewness (or more accurately Fisher's skewness) in the following formula:

$$\textbf{Skewness}\ S = \sum \left(\frac{r_i - \bar{r}}{\sigma_P} \right)^3 \times \frac{1}{n}$$

A normal distribution will have a skewness of 0. Note extreme values carry greater weight since they are cubed whilst maintaining their initial sign positive or negative.

Kurtosis (or more correctly Pearson's kurtosis[5]) provides additional information about the shape of a return distribution; formally it measures the weight of returns in the tails of the distribution relative to standard deviation but is more often associated as a measure of flatness or peakedness of the return distribution.

$$\textbf{Kurtosis}\ K = \sum \left(\frac{r_i - \bar{r}}{\sigma_p} \right)^4 \times \frac{1}{n}$$

The kurtosis of a normal distribution is 3; greater than 3 would indicate a peaked distribution with fat tails and less than 3 would indicate a less peaked distribution with thin tails. Extreme values carry even greater weight than skewness since the 4th power is used but negative and positive extreme events both make positive contributions.

A better understanding of the shape of the distribution of returns will aid in assessing the relative qualities of portfolios. Equity markets tend to have fat tails, when markets fall portfolio managers tend to sell and when they rise portfolio managers tend to buy, there is a higher probability of extreme events than the normal distribution would suggest. Therefore statistics calculated using normal assumptions might underestimate risk.

The mean is known as the first moment of the return distribution, variance or standard deviation the second moment, skewness the third moment and kurtosis the fourth moment. Investors should prefer high average returns, lower variance or standard deviation, positive skewness and lower kurtosis

Pezier[6] (2006) suggests using the Adjusted Sharpe Ratio which explicitly adjusts for skewness and kurtosis by incorporating a penalty factor for negative skewness and excess kurtosis as follows:

$$\textbf{Adjusted Sharpe Ratio} = SR \times \left[1 + \left(\frac{S}{6}\right) \times SR - \left(\frac{K-3}{24}\right) \times SR^2\right]$$

Predominately hedge fund management styles are designed to be asymmetric in their return patterns. If successful this leads to variability of returns on the upside but not on the downside. Investors are less concerned with variability on the upside but of course are extremely concerned about variability on the downside. This leads to an extended family of risk-adjusted measures reflecting the downside risk tolerances of investors seeking absolute not relative returns. .

Standard deviation and the symmetrical normal distribution are the foundations of Modern Portfolio Theory. Post-modern Portfolio Theory recognises that investors prefer upside risk rather than downside risk and utilises semi-standard deviation.

Semi-standard deviation measures the variability of underperformance below a minimum target rate. The minimum target rate could be the risk free rate, the benchmark or any other fixed threshold required by the client. All positive returns are included as zero in the calculation of semi-standard deviation or downside risk as follows:

$$\textbf{Downside Risk}\ \sigma_D = \sqrt{\sum_{i=1}^{n} \frac{\min\left[(r_i - r_T),0\right]^2}{n}}$$

Where:

r_T = Minimum Target Return

Downside potential is simply the average sum of returns below target:

$$\textbf{Downside Potential} = \sum_{i=1}^{n} \frac{\min\left[(r_i - r_T),0\right]}{n}$$

The equivalent upside statistics are as expected:

Upside Risk $\sigma_U = \sqrt{\sum_{i=1}^{n} \frac{\max\left[(r_i - r_T),0\right]^2}{n}}$

Upside Potential $= \sum_{i=1}^{n} \frac{\max\left[(r_i - r_T),0\right]}{n}$

In their article "A Universal Performance Measure" (2002) Shadwick & Keating [7] suggest a gain-loss ratio, *Omega* (Ω) that captures the information in the higher moments of a return distribution as follows:

$$\textbf{\textit{Omega} Ratio}\ \Omega = \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_i - r_T, 0)}{\frac{1}{n} \times \sum \max(r_T - r_i, 0)} = \frac{\text{Upside Potential}}{\text{Downside Potential}}$$

Omega ratio can be used as a ranking statistic, the higher; the better, it equals 1 when r_T is the mean return, it implicitly adjusts for both skewness and kurtosis in the return distribution.

The *Omega* ratio can also be converted to a ranking statistic in familiar form to the Sharpe ratio.

$$\textbf{Omega-Sharpe Ratio} = \frac{r_P - r_T}{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_T - r_i, 0)}$$

It can be shown that the Omega-Sharpe ratio is simply $\Omega - 1$ thus generating identical rankings to the *Omega* ratio [8]

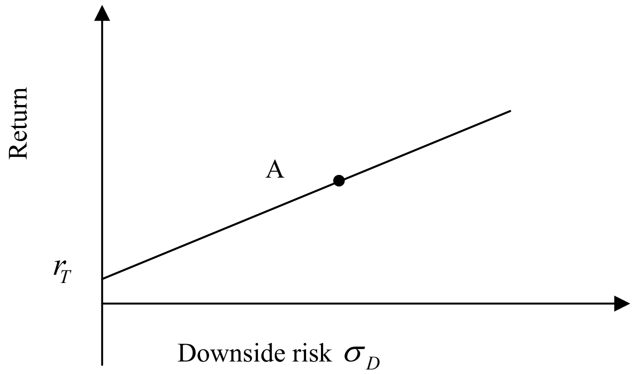
The Bernardo Ledoit [9] ratio (or Gain-Loss ratio) is a special case of the Omega ratio with $r_T = 0$

$$\textbf{Bernado Ledoit Ratio} = \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(r_i, 0)}{\frac{1}{n} \times \sum_{i=1}^{i=n} \max(0 - r_i, 0)}$$

A natural extension of the Sharpe and Omega-Sharpe ratios is suggested by Sortino [10] (1991) which uses downside risk in the denominator as follows:

$$\textbf{Sortino Ratio} = \frac{(r_P - r_T)}{\sigma_D}$$

Again graphically:



Total risk has simply been replaced by downside risk, portfolio managers will not be penalised for upside variability but will be penalised for variability below the minimum target return.

The upside potential ratio suggested by Sortino, Van de Meer & Platinga[11] (1999) can also be used to rank portfolio performance and combines upside potential with downside risk as follows:

$$\textbf{Upside Potential Ratio} = \frac{\sum_{i=1}^{i=n} \max(r_i - r_T, 0) \Big/ n}{\sigma_D} = \frac{\text{Upside Potential}}{\text{Downside Risk}}$$

Upside potential replaces the portfolio return above the target in the Sortino ratio. Notice the similarity to *Omega* except that performance below target is penalised further by using downside risk rather than downside potential.

Variability skewness[12] completes the transition from the *Omega* ratio utilising upside risk in the numerator

$$\textbf{Variability Skewness} = \frac{\text{Upside Risk}}{\text{Downside Risk}} = \frac{\sigma_U}{\sigma_D}$$

Watanabe[13] (2006) notes that people have a tendency to feel loss greater than gain a well known phenomena described by Prospect Theory[14], he suggests penalising loss as follows in the Prospect ratio

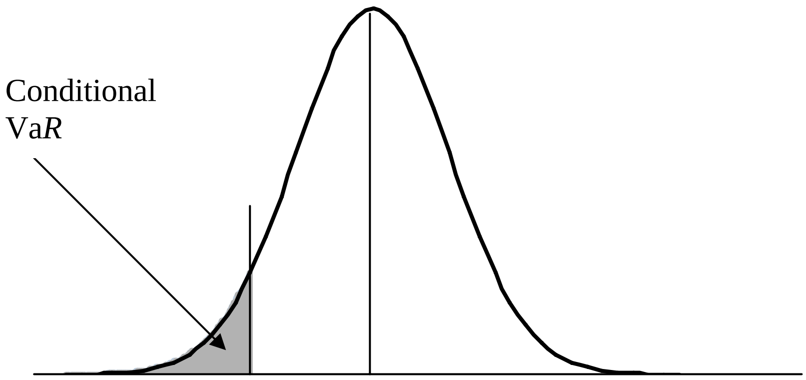
$$\textbf{Prospect Ratio} = \frac{\frac{1}{n} \times \sum_{i=1}^{i=n} \left(Max(r_i, 0) + 2.25 \times Min(r_i, 0)\right) - r_T}{\sigma_D}$$

If value at risk is your preferred measure of risk then, of course, there is a Sharpe type measure that uses Va*R* called reward to Va*R,* with Va*R* ratio (Va*R* expressed as a percentage of portfolio value rather than an amount) replacing standard deviation as the measure of risk in the denominator.

$$\textbf{Reward to VaR} = \frac{r_P - r_F}{\text{Va}R\ \text{Ratio}}$$

Va*R* does not provide any information about the shape of the tail or the expected size of loss beyond the confidence level. In this sense it is a very unsatisfactory risk measure; of more interest is Conditional Va*R* otherwise know as expected shortfall, mean expected loss, tail Va*R* or tail loss which takes into account the shape of the tail. Historical simulation methods which make no assumptions of normality are particularly suitable for calculating conditional Va*R*

Conditional Va*R*



Conditional Sharpe Ratio replaces Va*R* with Conditional Va*R* in the denominator of the Reward to Va*R* ratio. Clearly if expected shortfall is the major concern of the investor then the Conditional Sharpe Ratio is demonstrably favourable to the Reward to Va*R* ratio.

$$\textbf{Conditional Sharpe Ratio} = \frac{r_P - r_F}{\text{CVa}R}$$

Alternatively Va*R* can be modified to adjust for Kurtosis and Skewness using a Cornish-Fisher expansion as follows:

$$\text{MVa}R = \bar{r}_p + \left[z_c + \frac{z_c^2 - 1}{6} \times S + \frac{z_c^3 - 3z_c}{24} \times K_E - \frac{2z_c^3 - 5z_c}{36} \times S^2 \right] \times \sigma$$

Where:

z_c = -1.96 with 95% confidence

z_c = -2.33 with 99% confidence

This method works less well for distributions with more extreme skewness and excess kurtosis

Similar to the Adjusted Sharpe Ratio the Modified Sharpe Ratio uses Modified Va*R* adjusted for skewness and kurtosis.

$$\textbf{Modified Sharpe Ratio} = \frac{r_P - r_F}{\text{MVaR}}$$

Perhaps the simplest measure of risk in a return series from an absolute return investor's perspective, wishing to avoid losses, is any continuous losing return period or drawdown. The average drawdown is the average continuous negative return over an investment period, three years being a typical period of measurement for comparison purposes.

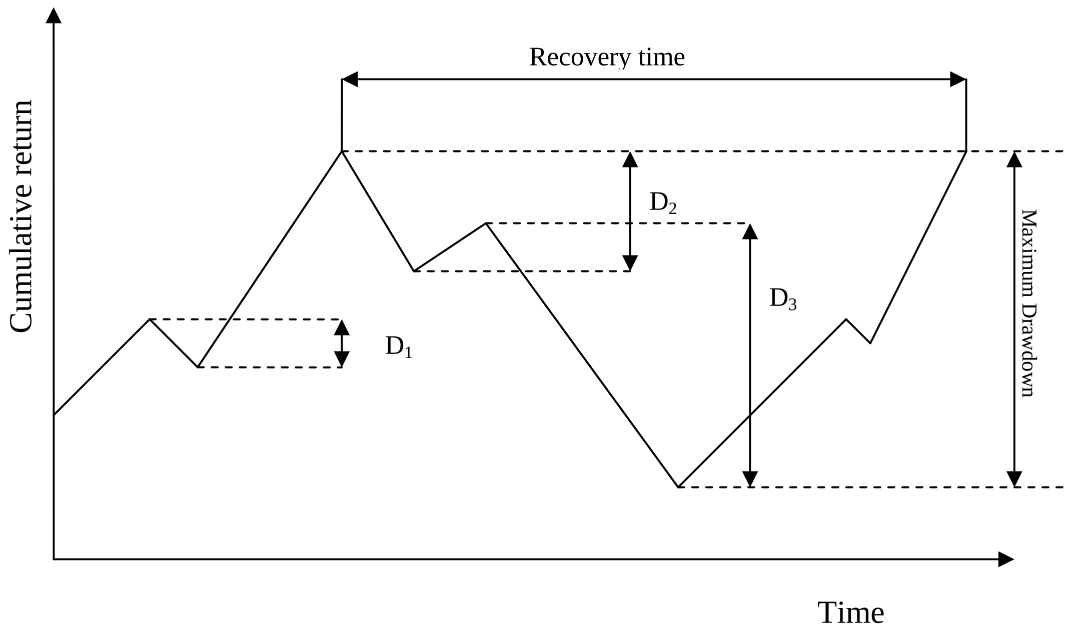
$$\textbf{Average drawdown } \overline{D} = \left| \sum_{j=1}^{j=d} \frac{D_j}{d} \right|$$

Where:

$D_j = j^{th}$ drawdown over entire period
d = total number of drawdowns in entire period

Some investors take the view that only the largest drawdowns in the return series are of any consequence and therefore restrict *d* to a predetermined maximum limit of say three or five thus enabling fair comparison between portfolios.

The maximum drawdown (D_{Max}), not to be confused with the largest individual drawdown, is the maximum potential loss over a specific time period, typically three years. Maximum drawdown represents the maximum loss an investor can suffer in the fund buying at the highest point and selling at lowest. Like any other statistic it is essential to compare performance over the same time period.



The Ulcer Index developed by Peter G Martin[15] in 1987 (so called because of the worry suffered by both the portfolio manager and investor) is similar to drawdown deviation with the exception that the impact of the duration of drawdowns is incorporated by selecting the negative return for each period below the previous peak or high water mark. The impact of long, deep drawdowns will have a significant impact since the underperformance since the last peak is squared.

$$\textbf{Ulcer Index} = \sqrt{\sum_{i=1}^{i=n} \frac{D_i'^2}{n}}$$

Where

$$D_i' = \text{drawdown since pervious peak in period i}$$

This approach is clearly sensitive to the frequency of time period and clearly penalises managers that take time to recovery to previous highs taking into account both the depth and duration of drawdowns.

If the drawdowns are not squared then the resulting Pain Index is very similar to the Zephyr Pain index in discrete form as proposed by Thomas Becker in 2006

$$\textbf{Pain Index} = \sum_{i=1}^{i=n} \frac{|D_i'|}{n}$$

The Calmar ratio (derived from **Cal**ifornia **M**anaged **A**ccount **R**eports) suggested by Terry Young[16] (1991) is a Sharpe type measure that uses maximum drawdown rather than standard deviation to reflect the investor's risk. In the context of hedge fund performance it is easy to understand why investor's might prefer the maximum possible loss from peak to valley as an appropriate measure of risk.

$$\textbf{Calmar Ratio} = \frac{r_P - r_F}{D_{Max}}$$

The risk free rate in the numerator is not a feature of the original definition but reflects the move from commodity and futures funds to traditional portfolio management. Arguably it should be included for all types of investors

The Sterling ratio replaces the maximum drawdown in the Calmar ratio with the average drawdown. There are multiple variations of the Sterling ratio in common usage, perhaps reflecting its use across a range of differing asset categories and outside the field of finance. The original definition attributed to Deane Sterling Jones[17] appears to be:

$$\textbf{Original Sterling Ratio} = \frac{r_p}{\overline{D}_{Lar} + 10\%}$$

The denominator is defined as the average largest drawdown plus 10%. The addition of 10% is arbitrary compensating for the fact that the average largest drawdown is inevitably smaller than the maximum drawdown. Typically only a fixed number of the largest drawdowns are averaged. With apologies to Deane Sterling Jones I suggest the definition is standardised to exclude the 10% but in Sharpe form as follows:

$$\textbf{Sterling Ratio} = \frac{r_p - r_F}{\left| \sum_{j=1}^{j=d} \frac{D_j}{d} \right|}$$

The number of observations *d* fixed to the investor's preference

Perhaps the most common variation of the Sterling ratio uses the average annual maximum drawdown in the denominator over three years. A combination of both Sterling and Calmar concepts, to avoid confusion and to encourage consistent use across the industry I suggest the following standardised definition[18]:

$$\textbf{Sterling-Calmar Ratio} = \frac{r_p - r_F}{\overline{D}_{\max}}$$

Given the variety of Sterling ratio definitions great care should be taken to ensure the same definition is used over the same time period using the same frequency of data when ranking portfolio performance.

Burke[19] (1994) in his article “A sharper Sharpe ratio” suggested using the familiar concept of the square root of the sum of the squares of each drawdown in order to penalise major drawdowns as opposed to many mild ones.

$$\textbf{Burke Ratio} = \frac{r_P - r_F}{\sqrt{\sum_{j=1}^{j=d} D_j^2}}$$

Just like the Sterling ratio the number of drawdowns used can be restricted to a set number of the largest drawdowns.

If the duration of drawdowns is a concern for investors the Martin ratio or *Ulcer Performance Index* is similar to the Burke Ratio but using the Ulcer Index in the denominator.

$$\textbf{Martin Ratio} = \frac{r_P - r_F}{\sqrt{\sum_{i=1}^{i=n} \frac{D_i'^2}{n}}}$$

The equivalent to the Martin ratio but using the Pain index is the Pain ratio.

$$\textbf{Pain Ratio} = \frac{r_P - r_F}{\sum_{i=1}^{i=m} \frac{D_i'}{n}}$$

With so many similar ratios the natural question to ask is “which is the best measure to use?” In fact Eling & Schuhmacher [20](2006) have published an article “Does the Choice of Performance Measure Influence the Evaluation of Hedge Funds” which concludes that most of these measures are all highly correlated and do not lead to significantly different rankings. Both the question and their article to some degree miss the point, risk like beauty is in the eye of the beholder, the investor most decide ex-ante which measures of return and risk best reflect their preferences and choose the combined ratio which reflects those preferences. One, and only one, of the above ratios are most likely to reflect the preferences of the investor.

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